Assignment 6, p. 1

1. a) $$\frac{1}{\tan x + \cot x} = \sin x \cos x$$

$$\frac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \sin x \cos x$$

$$\frac{1}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \sin x \cos x$$

$$\sin x \cos x = \sin x \cos x$$

b) $$\frac{\sec x - \cos x}{\sin x + \tan x} = \csc x - \cot x$$

$$\frac{\frac{1}{\cos x} - \cos x}{\sin x + \frac{\sin x}{\cos x}} = \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$\frac{\frac{1 - \cos^2 x}{\cos x}}{\frac{\sin x \cos x + \sin x}{\cos x}} = \frac{1 - \cos x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x(\cos x + 1)} = \frac{1 - \cos x}{\sin x}$$

$$\frac{\sin x}{\cos x + 1} = \frac{1 - \cos x}{\sin x}$$

$$\frac{\sin x}{\cos x + 1} = \frac{(1 - \cos x)(1 + \cos x)}{\sin x\,(1 + \cos x)}$$

$$\frac{\sin x}{\cos x + 1} = \frac{1 - \cos^2 x}{\sin x(\cos x + 1)}$$

$$\frac{\sin x}{\cos x + 1} = \frac{\sin^2 x}{\sin x(\cos x + 1)}$$

$$\frac{\sin x}{\cos x + 1} = \frac{\sin x}{\cos x + 1}$$

Assignment 6, p. 2

2. a) $\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = \cos\pi\cos\frac{\pi}{8} + \sin\pi\sin\frac{\pi}{8}$

$= \cos\pi\left(\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}\right) + \sin\pi\left(\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}\right)$

$= (-1)\left(\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}\right) + 0$

$= -\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}\cdot\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)} = -\sqrt{\frac{2-1}{4-2\sqrt{2}}} = -\frac{1}{\sqrt{4-2\sqrt{2}}}$

b) $\tan 105° = \tan(60° + 45°)$

$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$

$= \frac{1-3}{1-2\sqrt{3}+3}$

$= \frac{-2}{4-2\sqrt{3}}$

$= \frac{1}{\sqrt{3}-2}$

3. $4\cos^2\theta - 3 = 0$

$4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\frac{1}{2}\left((\cos(x+x)) + \cos(x-x)\right) = \frac{3}{4}$

$\cos 2x + 1 = \frac{3}{2}$

$\cos 2\theta = \frac{1}{2}$

let $z = 2\theta$

$\cos z = \frac{1}{2}$

$z = \frac{\pi}{3}, \quad z = \frac{5\pi}{3}$

$2\theta = \frac{\pi}{3}, \quad 2\theta = \frac{5\pi}{3}$

$\theta = \frac{\pi}{6}, \quad \theta = \frac{5\pi}{6}$

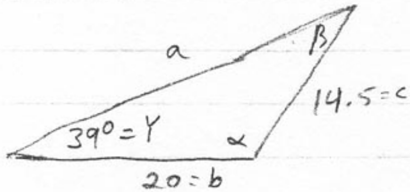
$\theta = \frac{\pi}{6} + n\pi, \quad \theta = \frac{5\pi}{6} + n\pi$ where n is any real integer.

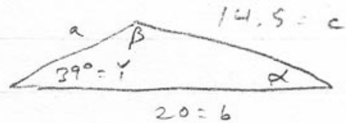
Assignment 6, p. 3

4. $\gamma = 39^\circ$

$b = 20$

$c = 14.5$





$$\frac{\sin 39^\circ}{14.5} = \frac{\sin \beta}{20} = \frac{\sin \alpha}{a}$$

$$\sin \beta = \frac{20 \sin 39^\circ}{14.5}$$

$$\sin \beta \simeq 0.868028125586$$

$\beta \simeq 60.2303^\circ$ or $\beta \simeq 119.7697^\circ$

$\alpha \simeq 180^\circ - 39^\circ - 60.2271^\circ \simeq 80.7697^\circ$ or $\alpha \simeq 180^\circ - 39^\circ - 119.7697 \simeq 21.2303$

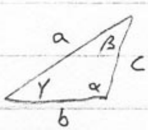
$$\frac{\sin 39^\circ}{14.5} \simeq \frac{\sin 80.7697^\circ}{a} \qquad \frac{\sin 39^\circ}{14.5} \simeq \frac{\sin 21.2303^\circ}{a}$$

$$a \simeq \frac{14.5 \sin 80.7697^\circ}{\sin 39^\circ} \qquad a \simeq \frac{14.5 \sin 21.2303^\circ}{\sin 39^\circ}$$

$a \simeq 22.7424$ $\qquad$ $a \simeq 8.3434$

---

(1)

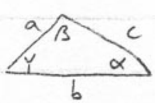


$a \simeq 22.7424$ $\qquad \alpha \simeq 80.7697^\circ$

$b = 20$ $\qquad \beta \simeq 60.2303^\circ$

$c = 14.5$ $\qquad \gamma = 39^\circ$

(2)



$a \simeq 8.3434$ $\qquad \alpha \simeq 21.2303^\circ$

$b = 20$ $\qquad \beta \simeq 119.7697^\circ$

$c = 14.5$ $\qquad \gamma = 39^\circ$

Assignment 6, p. 4

5.a) $a = 20$ $\quad c^2 = a^2 + b^2 - 2ab \cos y$

$b = 18$ $\quad 15^2 = 20^2 + 18^2 - 2(20)(18)\cos y$

$c = 15$ $\quad 225 = 400 + 324 - 720 \cos y$

$$720 \cos y = 724 - 225$$

$$\cos y = \frac{499}{720}$$

$$y \approx 46.1275^\circ$$

b) $A = \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{1}{2}(a+b+c)$$

$$s = \frac{1}{2}(20+18-15)$$

$$s = \frac{53}{2}$$

$$A = \sqrt{\frac{53}{2}\left(\frac{53}{2}-20\right)\left(\frac{53}{2}-18\right)\left(\frac{53}{2}-15\right)}$$

$$A = \sqrt{\frac{53}{2}\left(\frac{13}{2}\right)\left(\frac{17}{2}\right)\left(\frac{23}{2}\right)}$$

$$A = \sqrt{\frac{269399}{16}}$$

$$A = \frac{\sqrt{269399}}{4}$$

6. a) Polar: $\left(3, \frac{5\pi}{4}\right)$

$$r = 3$$

$$\theta = \frac{5\pi}{4}$$

$$x = r\cos\theta$$

$$x = 3\cos\frac{5\pi}{4}$$

$$x = \frac{-3}{\sqrt{2}}$$

$$y = r\sin\theta$$

$$y = 3\sin\frac{5\pi}{4}$$

$$y = -\frac{3}{\sqrt{2}}$$

Cartesian: $\left(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$

Check

$$\left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2 = 3^2$$

$$\frac{9}{2} + \frac{9}{2} = 9$$

$$\frac{18}{2} = 9 \checkmark$$

Assignment 6, p.5

6. b) Polar: $\left(-3\sqrt{2}, \frac{13\pi}{6}\right)$

$r = -3\sqrt{2}$

$\theta = \frac{\pi}{6}$

$x = r\cos\theta$

$x = -3\sqrt{2}\cos\frac{\pi}{6}$

$x = \frac{-3\sqrt{2}\sqrt{3}}{2}$

$x = -\frac{3}{2}\sqrt{6}$

$y = r\sin\theta$

$y = -3\sqrt{2}\sin\frac{\pi}{6}$

$y = -3\sqrt{2}\cdot\frac{1}{2}$

$y = -\frac{3}{2}\sqrt{2}$

Cartesian: $\left(-\frac{3}{2}\sqrt{6}, -\frac{3}{2}\sqrt{2}\right)$

check

$\left(\frac{3}{2}\sqrt{6}\right)^2 + \left(\frac{3}{2}\sqrt{2}\right)^2 = \left(3\sqrt{2}\right)^2$

$\frac{9\cdot 6}{4} + \frac{9\cdot 2}{4} = 9\cdot 2$

$\frac{72}{4} = 18$

$18 = 18$ ✓

7. a) Cartesian: $(-5, 0)$

$x = -5, \quad y = 0$

$r^2 = x^2 + y^2$

$r = \pm\sqrt{25}$

$r = \pm 5$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{0}{-5} = 0$

$\theta = \pi \quad \text{or} \quad \theta = 0$

$\therefore$ Polar: $(5, \pi + 2n\pi)$ or $(-5, 2n\pi)$ for any integer n.

Assignment 6, p 6

7. b) Cartesian: $(2\sqrt{3}, -6)$

$x = 2\sqrt{3}$, $y = -6$

$r^2 = (2\sqrt{3})^2 + (-6)^2$

$r = \pm\sqrt{12+36}$

$r = \pm\sqrt{48}$

$r = \pm 4\sqrt{3}$

$\tan\theta = \frac{-6}{2\sqrt{3}}$

$\tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$

Polar: $\left(4\sqrt{3}, \frac{5\pi}{3} + 2n\pi\right)$ or $\left(-4\sqrt{3}, \frac{2\pi}{3} + 2n\pi\right)$ for any integer n.

8. a) $a_2 = 4$ $a_5 = 13$

$4 + d + d + d = 13$

$3d = 9$

$d = 3$

$a_1 = a_2 - d$

$a_1 = 4 - 3$

$a_1 = 1$

$a_n = a_1 + (n-1)d$ for $n \geq 1$

$a_n = 1 + (n-1)3$

$\{3n-2\}_{n=1}^{\infty}$

b) $N = 2.737373\overline{73}$

$N = 2.\overline{73}$

$100N = 273.\overline{73}$

$99N = 271$

$N = \frac{271}{99}$

Alternate Method Using the Sum of a geometric Series

$N = 2.\overline{73}$

$N = 2 + \sum_{n=1}^{\infty} \frac{73}{100}\left(\frac{1}{100}\right)^{n-1}$

$N = 2 + \frac{\frac{73}{100}}{1 - \frac{1}{100}} = 2 + \frac{73}{100} \cdot \frac{100}{99} = 2 + \frac{73}{99}$

$N = \frac{271}{99}$